

flux in a load whose inductance is equal to the initial inductance of the previous deformed circuit, i.e., the factor is 2.86-1.75 for K of 0.75-0.95, respectively, this applying for each stage.

This method of generating a magnetic flux has been implemented in practice. Experiment has given an energy amplification coefficient of $0.9 \cdot 10^6$ and a flux increase coefficient of 310.

LITERATURE CITED

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MAXIMUM FORCE BETWEEN CURRENT CARRYING CONDUCTORS

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1. The ponderomotor interaction of current-carrying conductors is the basis of numerous technological applications. In a number of cases the question arises of the maximum possible interaction force per unit length between parallel cylindrical conductors, through each of which there flows a current I. For a specified current value this force depends on the form of the conductor cross section and the relative distance and configuration of the conductors, and can increase without limit if the conductor cross sections are sufficiently small and they are located sufficiently close to each other. However, such a technique of increasing ponderomotor force leads to an increase in conductor resistance and ohmic losses. Therefore it is of interest to determine the form of the conductor cross section for fixed conductor area S and uniformly distributed current density j which will maximize the force. This problem arose in optimizing the parameters of an electrodynamic vibrator used for vibroseismic sounding. The solution obtained was (Fig. 1) that the interaction force was maximal with each conductor having a semicircular cross section (naturally with currents flowing in opposite directions in each conductor there must be an insulation layer between the conductors, the thickness of which we will neglect in formulating the mathematical problem).

2. For simplicity, we will solve the problem with the assumption that the maximum interaction force F is achieved with the cross sections of the two conductors being identical, and these sections being located symmetrically with respect to the x and y axes (Fig. 2). The sections are described by the equation $x = \pm f(|y|)$, where $0 \leq y \leq a$ for the upper section and $-a \leq y \leq 0$ for the lower section. Each pair of elements dS_1 and dS_2 of the conductor cross sections interact producing a force the vertical component of which is equal to (notation as in Fig. 2)

$$dF = kdS_1 dS_2 \frac{\cos \theta}{r_{1,2}} = kdS_1 dS_2 \frac{y + |y'|}{(y + |y'|)^2 + (x - x')^2} \quad (1)$$

where $k = (\mu_0/2\pi)j_1 j_2$. Therefore the total interaction force, which is a function of f(y), can be written in the form

$$F[f(y), f(y')] = k \int_0^a dy \int_0^a dy' \int_{-f(y)}^{f(y)} dx \int_{-f(y')}^{f(y')} \frac{y + y'}{(y + y')^2 + (x - x')^2} dx' \quad (2)$$

It is necessary to find the maximum of this function with the additional condition

$$\int_0^a 2f(y) dy = S. \quad (3)$$

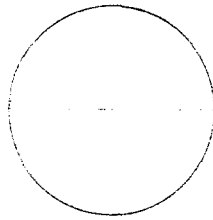


Fig. 1

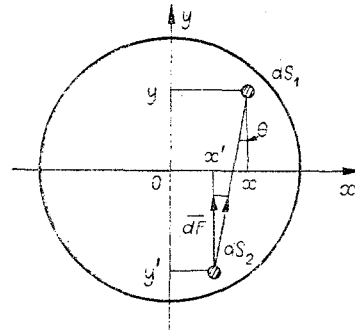


Fig. 2

By varying Eq. (2) with respect to f and considering Eq. (3), with the aid of undefined Lagrange factors, we obtain a nonlinear integral equation for f :

$$\int_0^a \left[\operatorname{arctg} \frac{f(y) + f(y')}{y + y'} + \operatorname{arctg} \frac{f(y') - f(y)}{y + y'} \right] dy' = \lambda. \quad (4)$$

The integrand of Eq. (4) has a simple geometric meaning. It is equal to the angle α over which the horizontal chord of the lower section lying at a distance y' from the x axis is visible from the point with ordinate y on the contour of the upper section. This permits us to guess one of the possible solutions of Eq. (4):

$$f(y) = \sqrt{a^2 - y^2}.$$

This corresponds to a semicircle, and fulfillment of Eq. (4) follows from the geometric fact that any chord is visible over one and the same angle from all points of the circumference.

Two further questions then arise: Is the solution found a local maximum of F , and are there other solutions corresponding to larger local maxima?

The first question may be answered by studying the second variation of Eq. (2) near the solution found:

$$\begin{aligned} \delta^2 F[f(y), f(y')] &= \frac{k}{a^2} \int_0^a \int_0^a \left[- \frac{\sqrt{a^2 - y^2} \sqrt{a^2 - y'^2}}{y + y'} (\delta f(y))^2 \right. \\ &\left. + \frac{2(a^2 + yy')}{y + y'} \delta f(y) \delta f(y') - \frac{\sqrt{a^2 - y^2} \sqrt{a^2 - y'^2}}{y + y'} (\delta f(y'))^2 \right] dy dy'. \end{aligned}$$

With consideration of the condition following from Eq. (3) on δf

$$\int_0^a \delta f(y) dy = 0$$

the expression for $\delta^2 F$ can be written in the form

$$\begin{aligned} \delta^2 F[f(y), f(y')] &= \frac{k}{a^2} \int_0^a \int_0^a \left[- \frac{\sqrt{a^2 - y^2} \sqrt{a^2 - y'^2}}{y + y'} ((\delta f(y))^2 \right. \\ &\left. + (\delta f(y'))^2) + \frac{2(a^2 + yy')}{y + y'} \delta f(y) \delta f(y') \right] dy dy' \\ &- \int_0^a \int_0^a \left\{ 2a \frac{y + y'}{y + y'} \delta f(y) \delta f(y') dy dy' \right\} = - \frac{k}{a^2} \int_0^a \int_0^a \frac{1}{y + y'} \{ [\sqrt{a^2 - y^2} \sqrt{a^2 - y'^2} \\ &- (a - y)(a - y')] ((\delta f(y))^2 + (\delta f(y'))^2) + (a - y)(a - y') (\delta f(y) - \delta f(y'))^2 \} dy dy', \end{aligned}$$

whence with consideration of the inequality

$$\sqrt{a^2 - y^2} > a - y \quad (0 < y < a)$$

it is immediately evident that $\delta^2 F < 0$, i.e., the solution found corresponds to a local maximum.

To answer the second question an algorithm was developed for maximization of Eq. (2) by the iteration method. (All solutions of the nonlinear integral equation (4) cannot be obtained analytically.) With various initial section shapes this algorithm invariably led to the solution found above. This permits the conclusion that we have obtained a complete solution.

Calculation of the maximum interaction force for the cross section determined here gives

$$F_{\max} = \frac{2\sqrt{2}}{3\pi^{3/2}} \mu_0 \frac{I^2}{\sqrt{S}} = 2.1277 \frac{I^2}{\sqrt{S}} \text{ (N/m)}.$$

For comparison, we note that for two circular conductors pressed against each other (of given section) the interaction force comprises 83% of the maximum value.

EXPERIMENTS ON INDUCTIVE-STORE CURRENT SWITCHING

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Considerable interest attaches to the scope for using inductive stores for systems of θ -pinch type with liners. A feature of the store operation in that case is the low inductance of the solenoid with the liner at the current-switching stage, which enables one to perform the switching with comparatively little energy loss.

One of the complicated problems is current interruption [1, 2]. Here we describe an inductive store with transformer output that can be used with liner compression, and we discuss results on the current switching. These experimental studies may be useful in the design of large systems with inductive stores.

Experimental Apparatus. The apparatus (Fig. 1) is a two-section inductive store ($L_{1.1}$, $L_{1.2}$); each section consists of 33 turns of diameter 660 mm made of aluminum rod of cross section 30×30 mm and connected in series. The sections have a center tap, which passes via the discharge gaps P_2 and P_3 to the ground line. The discharge gaps are adjusted for a breakdown voltage of ~ 5.5 kV and serve to prevent overvoltages on the turns arising if one or more of the trips does not operate or breaks down. The sets of trips (two for each winding) are connected in series with the sections. The overall inductance of the primary winding is $5 \cdot 10^{-4}$ H. The store is supplied by the discharge current from the capacitor bank C_1 with a total stored energy of 1.2 MJ at a maximum voltage of 5 kV. The maximum charging current is ~ 60 kA, which corresponds to an energy of ~ 1 MJ. The capacitor bank is connected to the store by the mechanical trip P_1 with solid insulator.

The energy output to the load is by means of a transformer circuit with parallel connection of the turns in the secondary winding, which consists of two turns of diameter 690 mm and length 10^3 mm, each of which encompasses one section. The turns are made of St.3 steel of thickness 8 mm with a collector for connecting the cables. The current in the secondary winding (after opening the primary circuit) passes via a cable feeder to the load. The accelerating solenoid is of steel and consists of one turn with an internal diameter of 150 mm and a length of 120 mm. We examined only the current switching in these experiments, so a metal insert of $\varnothing 148$ mm was used instead of the liner.

The store operates as follows: after P_1 has operated (Fig. 1), the charging begins. When the necessary current is reached, the trips K_1 - K_4 operate and the current is diverted to the wires F_1 - F_4 , which explode and open the circuit of the primary winding, and then a multiplied current appears in the secondary winding. To equalize the voltages on the trips, each is connected to a sectional resistor R_1 - $R_4 = 3 \Omega$ in such a way that each trip is shunted by a resistance of 1Ω .

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